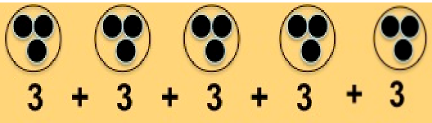
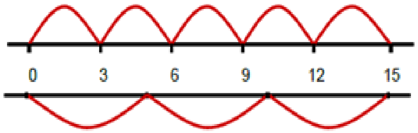
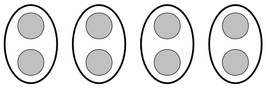
Exemplar progression in the written calculation of **MULTIPLICATION**

Year	Calculation method and examples	Guidance
R-1	Previous to Year 2, children should work using repeated addition (refer to the addition section of this policy), pictorial representation and kinaesthetic methods including Numicon, bead strings, counting equipment and also record in the standard notation for multiplication relating to times tables. Learners will be encouraged to generate their own pictorial representation. **Counting will be practical** and addition will be shown in sets resembling arrays and drawn on open number lines five **<u>sets</u>** of three: 5 x 3 = 15  Discuss the commutative law. Therefore 3 x 5 or 5 x 3  Count with money in denominations  Pictures and objects  We have 6 cakes put 2 on each plate. 3 plates, two cakes on each plate Real life counting activities e.g. pairs of shoes, pairs of socks, groups of fingers/toes, legs on a dog etc…	*Introduce the operation x* *Introduce arrays* *Introduce repeated addition* *Introduce number bonds* *Introduce times tables 2, 5, 10* *Introduce doubling* *Notation can be introduced by teachers from Year 1(When is at the teachers' discretion).*

2 - 3

Heavy emphasis is placed on times table learning – 2, 5 & 10 in year 2 followed by 3, 4 & 8 in year 3.

Arrays showing links to division which should be accompanied by <u>CONCRETES</u>



Reinforcement of multiplying by 10 and 100 and the effect this has on the place value of the digits using concretes and/or ICT visualised models to support.

6 X 10 = 60
6 X 100 = 600
6 X 20 = 6 X 2 X 10 = 120

Children are expected to make arrays using whatever classroom concretes are available – Deinnes, counters, unifix blocks, lumps of cheese etc…

3 – 4
(Later in Year 3)

Grid Method of multiplication - multiplying by 1 digit and conforming to the times table requirements of the year group (2's, 3's, 4's, 5's, 8's, 10's)

37 x 8 = 259

x	30	7	
8	240	56	**296**

Extend into bigger numbers using grid layout

238 x 4

x	200	30	8	
4	800	120	32	**952**

This method will be the school's first expected method - introduction of standard written methods occur later in school.

It should increase in difficulty throughout the child's journey within school and should be secure enough to offer a fall-back for those children who may struggle to grasp standard methods.

<table>
<tr>
<td>5</td>
<td>
<p>Grid Method of multiplication - multiplying by 2 digit and increasing use in problem solving involving multiplying with all 12 times tables up to 12x.</p>
<p>56 x 27</p>
<table>
<tr><td>x</td><td>50</td><td>6</td><td></td></tr>
<tr><td>20</td><td>1000</td><td>120</td><td>1120</td></tr>
<tr><td>7</td><td>350</td><td>42</td><td>392</td></tr>
<tr><td></td><td></td><td></td><td>1512</td></tr>
</table>
<p>Teaching of grid method will of course involve a secure knowledge of related facts (i.e. 2 x 5 = 10 therefore 20 x 5 = 100 and 200 x 5 = 1000) – this will affect the level of problem posed and must be taken into account when planning and setting challenge.</p>
</td>
<td>
<p>Children MUST understand why we need to multiply each digit by each other digit and is should always at this stage, involve CONCRETES.</p>
<p>Inability to use mental calculation to solve the final addition should fall back to the written calculation strategy for addition in Year 5 (see section detailing addition).</p>
<p>Some children may be encouraged to add in two smaller groups and then undertake a further addition to find the final answer:</p>
<p>1000
350
120
42</p>
<p>Or</p>
<table>
<tr><td>1000
350</td><td>120
42</td><td>1350
162</td></tr>
<tr><td>1350</td><td>162</td><td>1512</td></tr>
</table>
</td>
</tr>
<tr>
<td>5 & 6</td>
<td>
<p>Grid Method of multiplication extending into the multiplication of decimal numbers involving all times tables up to 12x.</p>
<p>5.24 x 6</p>
<table>
<tr><td>x</td><td>5.00</td><td>0.2</td><td>0.04</td><td></td></tr>
<tr><td>6</td><td>30</td><td>1.2</td><td>0.24</td><td>31.44</td></tr>
</table>
</td>
<td>
<p>Inability to use mental calculation to solve the final addition should fall back to the written calculation strategy for addition in Year 5 (see section detailing addition).</p>
</td>
</tr>
</table>

EXTENSION INTO VERTICAL FORMAT WITH STANDARD SHORT AND LONG METHODS OF MULTIPLICATION – avoiding adding over 10/100/1000 when adding to the carried number which complicates the matter.

STANDARD SHORT MULTIPLICATION

(multiplying by 1 digit)

24 x 6

$$\begin{array}{rccc} & & 2 & 4 \\ \times & & & 6 \\ \hline & 1 & 4 & 4 \\ \hline & & 2 & \end{array}$$

242 x 7

$$\begin{array}{rcccc} & & 2 & 4 & 2 \\ \times & & & & 7 \\ \hline & 1 & 6 & 9 & 4 \\ \hline & & 2 & 1 & \end{array}$$

In explanation of this method, children’s awareness should be drawn to the similarities to the Grid Method and that this requires less recording.

Teaching may indeed begin by instruction in both methods simultaneously to highlight the comparison.

Children chosen to progress to this method should be carefully selected and the method should indeed be withdrawn if the teacher at any time feels that it is causing confusion. – grid method of multiplication should then be re-focussed upon

STANDARD LONG MULTIPLICATION

(multiplying 2-digit by 1 digit)

24 x 16

$$\begin{array}{cccc} & & 2 & 4 \\ \times & & 1 & 6 \\ \hline & 1 & 4 & 4 \\ & & {\scriptstyle 2} & \end{array}$$

$$\begin{array}{cccc} & & 2 & 4 \\ \times & & 1 & 6 \\ \hline & 1 & 4 & 4 \\ & & {\scriptstyle 2} & \\ & & & 0 \end{array}$$

$$\begin{array}{cccc} & & 2 & 4 \\ \times & & 1 & 6 \\ \hline & 1 & 4 & 4 \\ & & {\scriptstyle 2} & \\ & 2 & 4 & 0 \\ \hline & 3 & 8 & 4 \\ \hline \end{array}$$

(multiplying 3-digit by 2-digit)

$$\begin{array}{cccc} & 1 & 2 & 4 \\ \times & & 2 & 6 \\ \hline & 7 & 4 & 4 \\ & {\scriptstyle 1} & {\scriptstyle 2} & \\ 2 & 4 & 8 & 0 \\ \hline 3 & 2 & 2 & 4 \\ \hline {\scriptstyle 1} & {\scriptstyle 1} & & \end{array}$$

STEP-BY-STEP

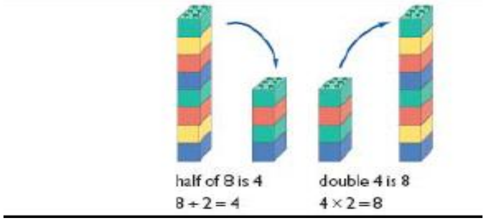
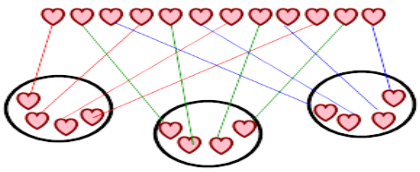
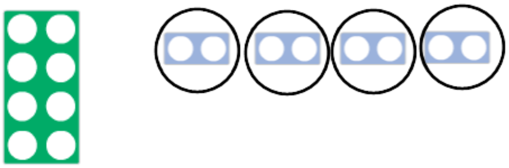
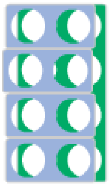
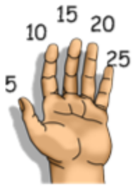
1. Multiply bottom unit by top unit noting the ten as a carried number in anticipation of the second multiplication step;
2. Multiply bottom unit by top ten (note: the 120 product of multiplying 20 by 6 is added to the carried 20 before recording);
3. Place a zero in the units column because we are now multiplying by a multiple of ten;
4. Multiply the bottom ten by the top unit to make 40;
5. Multiply the bottom unit by the top unit to make 200;
6. Perform a standard columnar addition to find the final answer.

Following the same pattern as before multiplying first the bottom unit by the top unit, ten and hundred followed by the bottom ten multiplied by the top unit, ten and hundred.

NOTE

In variant methods the carried numbers used in the multiplication phase of the problem are carried above to avoid confusion when adding.

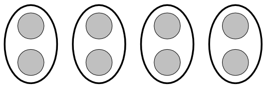
Exemplar progression in the written calculation of **Division**

Year	Calculation method and examples	Guidance
R - 2	Children will be taught division through the use of visual and kinaesthetic grouping, sharing and chunking. **Object and picture examples** Numicon, drawings, counters, Deinnes or other: **Halving**  12 ÷ 3 =4 12 shared between 3 How many **SETS** of three are in twelve?  How many **SETS** of 2 are in 8? 8 ÷ 2 = 4  Or  25 ÷ 5 = 	<u>New Vernacular:</u> Use of the word **SETs** no longer groups or lots. In R and the initial stages of Y1 + and ÷ are not used. This is preceeeded by continuous verbal and practical maths.

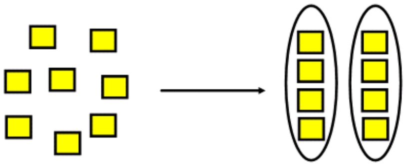
2-3

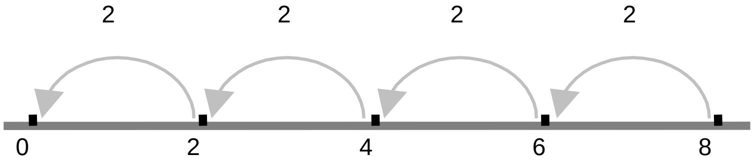
Number lines and **Diagrammatic grouping**

As in previous years, children should be aided by concretes to remove **SETS** and help create visual models of their problems. In Year 2, abstract division should be preceded by division with image or concrete aid to reinforce the essence of division.



8 ÷ 2 = **4 (Using arrays above and using both Deinnes and a number-line below)**





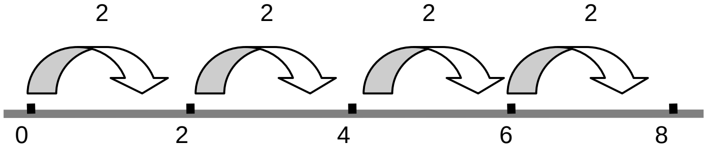
The use of the number line is to stress the relationship between repeated subtraction and division.

Division by repeated subtraction is represented by counting back and counting up.

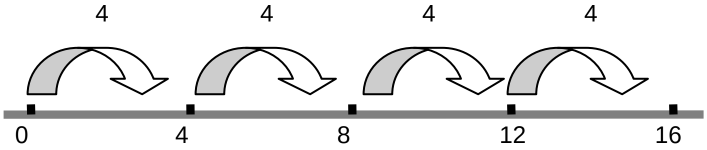
While Year 3 represents a move toward written division strategies, the abstract notion of division is often very difficult for children to grasp and should therefore continue to be taught with the aid of counters or interactive programmes to reinforce the notion.

CHUNKING – remove chunks to reinforce the repetitive subtraction – supported of course, by concretes

8 ÷ 2 = 4



16 ÷ 4 = 6



If children are working with 2's, 5's and 10 times tables in Year 2, then they can be exposed to division as the inverse through pictorial representation and standard notation.

In Year 3, the same applies when working with 3, 4 and 8 x tables.

4

Missing number and recording of known facts relating to multiplication

$4 \times 6 = 24$

$\boxed{} \times 7 = 21$

$3 \times \boxed{} = 32$

$36 \div 4 = \boxed{}$

$\boxed{} \div 5 = 7$

$24 \div \boxed{} = 6$

Standard notation of division introduced when working with known times table facts – 2, 5, 10, 3, 4, 8

(no remainders – still using concretes to show this - Deinnes)

$$\begin{array}{r} 9 \\ 5\overline{)\;4\;5} \end{array} \qquad \begin{array}{r} 8 \\ 10\overline{)\;8\;0} \end{array} \qquad \begin{array}{r} 6 \\ 6\overline{)\;3\;6} \end{array}$$

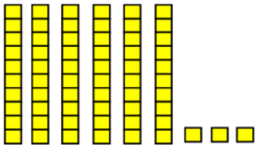
There should be no discussion of short (or bus stop) division. Simply an alternate way of recording a division sum and it's answer (as an alternative to 45 ÷ 5 = 9)

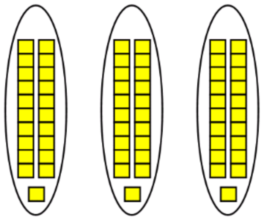
Informal methods using multiples of the divisor or 'chunking'

TU ÷ U (including remainders)

63 ÷ 3 = 21 – Children need to make the number and split into 3 SETS, exchanging tens for ones when they need to:

Initiall,y CHUNKING should be taught as *chunking up* from 0 to the target number relating addition to subtraction and therefore division. Chunking down will follow in year 5





This method can be recorded in the following manner $\begin{array}{r} 21 \\ 3\overline{)\;63} \end{array}$

It can also be linked to number families:

63 ÷ 3 = 21

63 ÷ 21 = 3

3 x 21 = 63

21 x 3 = 63

	The above method will link into the instruction of chunking as the first step in the extended written calculation of division, beginn**ing with none-remainder** problems and progressing to the use of remainders. 72 ÷ 5 = **14 r 2** This method should be introduced alongside Deinnes to show the repetitive SETS of 5 that are being removed $\begin{array}{r\|l} & \mathbf{1\ 4\ \ r\,2} \\ \hline 5 & 7\ 2 \end{array}$ + 5 0 (**10** x 5) + 2 0 (**4** x 5) **7 0**	If it is helpful, children should be encouraged to write lists of multiples at the side of their written calculations to aid them. This may also form part of the modelling of the strategy: **50** **45** **40** **35** **30** **25** **20** **15** **10** **5**
5	**Informal methods using multiples of the divisor or 'chunking'** **HTU ÷ U** (including remainders) As in Year 4, but introducing CHUNKING DOWN as an alternative strategy, reinforcing the link between division and repeated subtraction. As methods become entrenched, class teachers should extend the use to 3-digit numbers.	Children will choose their own favoured method (chunking either up or down) but both should be modelled by the class teacher.

	2 5 6 ÷ 7 = **3 6 r 4** Chunking down using subtraction **3 6 r 4** $7\overline{)\,2\,5\,6}$ - 2 1 0 (**30**x7) 4 6 - 4 2 (**6**x7) **4** Chunking up using addition **3 6 r 4** $7\overline{)\,2\,5\,6}$ 0 + 2 1 0 (**30**x7) + 4 2 (**6**x7) 2 5 2	
6	**Informal methods using multiples of the divisor or 'chunking'** **HTU ÷ U & TU** (including remainders and decimals) ***HTU ÷ U*** 5 6 0 ÷ 2 4 = **2 3 r 8** **2 3 r 8** $24\overline{)\,5\,6\,0}$ - 4 8 0 (**20** x 24) 8 0 - 7 2 (**3** x 24) **8** As understanding deepens, remainders become fraction – 8/24 and children are able to simplify to 1/3	In Year 6 chunking should be done upwards (using addition) or downwards (using subtraction) in accordance with the students' personal preference. Teachers however, should continue to model both styles.

Decimal division

87.5 ÷ 7 = **12.5**

1 2 . 5

7 | 8 7 . 5

\- 7 0 (**10** x 7)

1 7 . 5

\- 1 4 (**2** x 7)

3 . 5

\- 3 . 5 (**0.5** x 7)

0

Work on dividing decimals should certainly be preceded by work concentrating on known multiplication facts i.e. 5 x 7 = 35 therefore 500 x 7 = 3500, 50 x 7 = 350 and 0.5 x 7 = 3.5

EXTENSION INTO STANDARD METHOD – BOTH SHORT AND LONG METHODS OF DIVISION

5 – 6

Short Division (All introduced throughout Year 5)

(Stage 1) 98 ÷ 7 = 14

1 4

7 | 9 ^{2}8

Answer: 14

Script:
How many 7s in 90? *10*. So the 1 goes above, in the tens column. How many remaining? *20*. This is placed here, just like when we exchange in subtraction. How many 7s are there in 28? *4*. Record it here above the units/ones column because it is a single digit answer

	(Stage 2) 423 ÷ 5 = 86 r2 $\begin{array}{r\|lll} & 8 & 6 & r2 \\ \hline 5 & 4 & 3 & {}^{3}2 \end{array}$ Answer: 86 r2 (Stage 3) 496 ÷ 11 = 45 r1 $\begin{array}{r\|lll} & 4 & 5 & r1 \\ \hline 11 & 4 & 9 & {}^{5}6 \end{array}$ Answer: 45 r1 leading to 45 $\frac{1}{11}$	**Script:** How many 5s are there in 430? Well, there are 8 5s in 40, so 80 in 400. *80*. The answer goes here, in the tens column. There are 30 left over so we can record it here, just like when we exchange in subtraction. Now, how many 5s are there in 32? *6*. And how many remain? *2*
6	**Long Division (All introduced to higher level children following chunking detailed for level 6)** $\begin{array}{r\|lllll} & & 2 & 8 & . & 8 \\ \hline 15 & 4 & 3 & 2 & . & 0 \\ & \underline{3} & \underline{0} & \downarrow & & \downarrow \\ & 1 & 3 & 2 & & \\ & \underline{1} & \underline{2} & \underline{0} & & \\ & & 1 & 2 & & 0 \\ & & \underline{1} & \underline{2} & & \underline{0} \\ & & & & & 0 \end{array}$	This method will only be introduced to a section of more able children at the discretion of the current Year 6 teaching team. It may be used by children who have been taught this method at home – if this is the case, then the school will support this method if it being used successfully. It may also be suggested for any children who are still unable to grasp chunking, but whose further use and knowledge of Maths suggests that an alternate approach may be needed.